B.Sc.III – MATHEMATICS (PAPER–THIRD), 2014

(NUMERICAL ANALYSIS AND PROGRAMMING IN C)

Time : Three Hours *Maximum Marks : 75*

Note : Attempt questions from all the Sections.

SECTION-A

(SHORT ANSWER TYPE QUESTIONS)

Note : Attempt any ten questions. Each question carries 5 marks. **(3×10=30)**

1. If Δ is a forward difference operator and ∇ is a backward difference operator. Show that

   (i) $(1+\Delta)(1-\nabla)=1$

   (ii) $D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+.....)$

   Where D is the differential operator & h is interval of differencing.

2. Find the function whose first difference is $9x^2+11x+5$

3. Evaluate $\Delta^n\left(\frac{1}{x}\right)$.

4. If δ is the central difference operator and μ is the average operator. Prove that $\mu^2=1+\frac{\delta^2}{4}$.

5. The following table is given

x	0	1	2	3	4
$f(x)$	3	6	11	18	27

   What is the form of the following $f(x)$?

6. Prove that

$$\frac{d}{dx}(y_x)=\frac{1}{h}(y_{x+h}-y_{x-h})-\frac{1}{2h}(y_{x+2h}-y_{x-2h})+\frac{1}{3h}(y_{x+3h}-y_{x-3h})+...$$

7. Evaluate $\int_0^6\frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}rd$ rule and Simpson's $s\frac{3}{8}th$ rule.

8. Apply Given's method to reduce the following matrix to tri-diagonal form $A=\begin{bmatrix}2&1&3\\1&4&2\\3&2&3\end{bmatrix}$.

9. Define recursion. Write a program in C language using recursion to find factorial of a number.
10. Solve the following initial value problem by the generating function technique $y_{h+2} - 5y_{h+1} + 6y_h = 2$ if $y_0 = 1$, $y_1 = 2$.
11. Describe Picards method of successive approximations.
12. Prove that $\Delta^2 0^m = n\Delta^{n-1}0^{m-1} + n\Delta^n 0^{m-1}$ where $\Delta^n x^m |_{x=0} = \Delta^n 0^m$
13. By using Newton-Raphson method, Find the root of $x^4 - x - 10 = 0$ which is nearer to x = 2, correct to three places of decimal.
14. Define the following statement in C-language.
    (i) If statement and if-else statement.
    (ii) Switch Statement
15. Obtain the Chebyshev Linear polynomial approximation to the function $f(x) = x^3$ on [0, 1].

SECTION-B
(LONG ANSWER TYPE QUESTIONS)

Note : Attempt any three questions. Each question carries 15 marks.

(15×3=45)

1. (a) Use method of Separation of Symbols to Prove that

$$u_1 x + u_2 x^2 + u_3 x^3 + \ldots = \frac{x}{1-x}u_1 + \frac{x^2}{(1-x)^2}\Delta u_1 + \frac{x^3}{(1-x)^3}\Delta^2 u_1 + \ldots$$

(b) If p,q,r,s be the successive entries corresponding to equidistant arguments in a table, show that when third difference are taken into account, the entry corresponding to the argument half-way between arguments of q and r is $A + \frac{1}{24}B$, where A is arithmetic mean of q and r, B is the arithmetic mean of $3q - 3p - s$ and $3r - 2s - p$.

2. (a) Four equidistant values u_{-1} u_0, u_1 and u_2 being given, a values is interpolated by Lagrange's. Show that it may be written in the form $u_x = yu_0 + xu_1 + y\frac{(y^2-1)}{3!}\Delta^2 u_1 + \frac{x(x^2-1)}{3!}\Delta^2 u_0$. Where $x + y = 1$.

(b) Apply Bessd's formula to find y_{25} Given $y_{20} = 24, y_{24} = 32, y_{28} = 35, y_{32} = 40$.

3. (a) Obtain the cube root of 12 to five decimal places by Newton Raphson method.

(b) Find f′ (5) from the following table

X	0	2	3	4	7	9
$F_{(x)}$	4	26	58	112	466	922

4. (a) If $u_x = a + bx + cx^2$, Prove that

$\int_1^3 u_x dx = 2u_2 + \frac{1}{12}(u_0 - 2u_2 + u_4)$ and hence find an approximate value for $\int_{-1/2}^{1/2} e^{-x^{2/20}} dx$.

(b) Use Picard's method to approximate y when $x = 0.2$ given that $y = 1$ when $x = 0$ and $\frac{dy}{dx} = x - y$.

5. (a) Using Jacobi's method, find the eigen values of the matrix A given by $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$.

(b) Solve the following difference equations.

$y_{h+2} - 4y_{h+1} + 4y_h = 3h + 2^h$

6. Write short notes on any Two of the following :

(a) Runga-Kutta method for solving differential equation second order and third order.

(b) QR-Method for finding eigen values.

(c) Legendre's Polynomials

(d) Decision statements and control structures in C-language with examples.

(e) Language's Interpolation formula.

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