

BUNDELKHAND UNIVERSITY, JHANSI
B.Sc. (Part-III) MATHEMATICS (Paper-II), 2015

(COMPLEX ANALYSIS)

Time: Three Hrs.

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M.M. : 75

Note : Attempt question from all the sections.

सभी खण्डों से प्रश्नों के उत्तर दीजिए।

Section -A खण्ड-अ

(Short Answer Type Questions) (लघु उत्तरीय प्रश्न)

Note : Attempt any ten questions. Each question carries 3 marks. $10 \times 3 = 30$

किन्हीं दस प्रश्नों के उत्तर दीजिये। प्रत्येक प्रश्न 3 अंकों का है।

1. Show that the function $f(Z) = u + iv$, UPadda.com

where $f(Z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, z \neq 0$

UPadda.com $= 0, (z=0)$

is continuous for all values of z .

- Show that the function $f(Z) = z|z|$ is not analytic anywhere.
- If $f(z) = u + iv$ is an analytic function, show that u and v are both harmonic functions.
- Consider the transformation $\omega = e^{ix/4}$, and determine the region D' in the ω -plane in which the region D bounded by the lines $x = 0, y = 0$ and $x + y = 1$ in the z -plane is mapped.
- Find the image of the infinite strip $0 < y \leq \frac{1}{2}$ under the transformation $\omega = \frac{1}{z}$ and show the regions graphically with proper interpretation.
- Let $\omega = T_1(z) = \frac{z+2}{z+3}$ and $\omega = T_2(z) = \frac{z}{z+1}$ be two transformations, then find (1) $T_1^{-1}(\omega)$ and $T_2 T_1(z)$.
- Define (i) Fixed point (ii) critical points with examples thereof.
- Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along the real axis from $z = 0$ to $z = 2$ and then along a line parallel to y -axis from $z = 2$ to $z = 2 + i$.
- State Cauchy's inequality and establish it.
- Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$, if C is the curve $|z+1| = 1$.

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11. Expand $\frac{1}{z^2 - 3z + 2}$ in the region $|z| < 2$. UPadda.com

12. Find the nature of singularities of the function $f(z) = \frac{z - \sin z}{z^3}$.

13. Find the residue of $f(z) = \frac{z^3}{z^2 - 1}$ at $z = \infty$ UPadda.com

14. Evaluate $\int_C \frac{2z^2 + 5dz}{(z+2)^3(z^2+4)}$, where C is the square with vertices at $1 + i, 2+i, 2+2i, 1+2i$.

15. Find the image and draw a rough sketch of the mapping of the region $1 \leq x \leq 2$ and $2 \leq y \leq 3$, under the mapping $\omega = e^z$.

Section -B खण्ड-ब

(Long Answer Type Questions) (दीर्घ उत्तरीय प्रश्न)

Note : Attempt any three questions. Each question carries 15 marks. $15 \times 3 = 45$

किन्हीं तीन प्रश्नों के उत्तर दीजिये। प्रत्येक प्रश्न 15 अंकों का है।

- Define an analytic function $f(z)$. What are the necessary and sufficient conditions for the function $f(z)$ to be analytic, prove it.
- Evaluate $\int_0^\pi \frac{a d\theta}{a^2 + \sin^2 \theta}, a > 0$ using contour integration method.
- Show that the positive half of the ω plane given by $v \geq 0$ corresponds to the circle $|Z| \leq 1$ in the z -plane under the transformation $z = \frac{i - \omega}{i + \omega}$. Also interpret: $-u \geq 0$ implies what with proper explanation.
- If $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z . UPadda.com
- Apply the calculus of residues to prove that $\int_0^\infty \frac{\sin \pi x}{x(1-x^2)} dx = \pi$

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