B.Sc.III – MATHEMATICS (PAPER–SECOND), 2014
(ABSTRACT ALGEBRA)

Time : Three Hours — *Maximum Marks : 75*

Note : Attempt questions from all the Sections.

SECTION-A
(SHORT ANSWER TYPE QUESTIONS)

Note : Attempt any six questions. Each question carries 5 marks.

(5×6=30)

1. Define group. Also show that the mapping f : I→I, such that $f(x) = -x \forall x \in I$, is an auto morphism of the additive group of integers I.

2. Let G be a group, H is a subgroup of G, f is an auto morphism of G. Let
$8(H) = \{f(h) : h \in H\}$
Prove that $f(H)$ is a subgroup of G.

3. Prove that the intersection of two ideals of a ring R is again on deal of R.
4. Let G be a finite cyclic group of order n. Determine Automorphism G the group of all automorphisms of G.
5. Prove that the relation of conjugacy is an equivalence relation on G.
6. Define commutative ring. Give an example
7. In a ring R prove that –(–a) = a
8. Prove that every field is an Euclidean Ring.
9. Show that ring of integers is a principal ideal ring.
10. Define Vectors space.
11. A linear transformation T is definied on $V_2(c)$ by $T(a,b) = (\alpha a + \beta b, \gamma a + \delta d)$ where $\alpha, \beta, \gamma, \delta$ are fixed elements of C. Prove that T is invertible iff $a\delta - \beta\gamma \neq 0$
12. **Define linear transformation.**

SECTION-B

(LONG ANSWER TYPE QUESTIONS)

Note : Attempt any three questions. Each question carries 15 marks.

(15×3=45)

1. Let U be an n-dimensional vector space over the field F, and let V be an m-dimensional vector space over F. Then prove that the vector space L (U,V) of all linear transformations from U into V is finite dimensional and is of dimension m n.

2. Show that the mapping $a \to a^{-1}$ is an auto morphism of a group G if and only if G is abelian.

3. Show that a finite commutative ring without zero divisors is a field.

4. Show that any ring R without a unity element can be imbedded in a ring with unity.

5. If D is an integral domain, then the polynomial ring D[x] is also an integral domain.

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