BUNDELKHAND UNIVERSITY, JHANSI

B.Sc.II – MATHEMATICS (PAPER–FIRST), 2015

(LINEAR ALGEBRA AND MATRICES)

Time : Three Hours *Maximum Marks* : 33

Note : Attempt questions from all the Sections.
सभी खण्डों से प्रश्नों के उत्तर दीजिए।

SECTION - A (खण्ड-अ)

(SHORT ANSWER TYPE QUESTIONS) (लघु उत्तरीय प्रश्न)

Note : Attempt any six questions. Each question carries 2 marks **(2×6=12)**

किन्हीं छः प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न 2 अंकों का है।

1. Show that

$$A = \begin{bmatrix} i & 3+2i & -2-i \\ -3+2i & 0 & 3-4i \\ 2-i & -3-4i & -2i \end{bmatrix}$$

is skew hermitian matrix.

2. Prove that the following sets of vectors of R^3 is linearly dependent
(1, 0, 0), (0, 1, 0), (1, 1, 1) (–1, 1, –1)

3. Find the rank of a matrix

$$\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$$

4. Let W be the subspace of R^4 generated by the vectors (1, –2, 5, –3), (2, 3, 1, –4) and (3, 8, –3, –5). Find a basis and the dimension of W.

5. Find the characteristic root of the matrix

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

6. Let the following subsets of F^3

$w_1 = \int(x_1, x_2, x_3) \int x_1 + x_2 + x_3 = 1 \int$

$w_2 = \int(y_1, y_2, y_3) \int y_1 + y_2 + y_3 = 1 \int$

Prove that w_1 is a subspace of F^3 and w_2 is not a subspace of F^3.

7. Solve the simultaneous equations

$x + y + z = 3$
$x + 2y + 3z = 4$
$x + 4y + 9z = 6$

8. Prove that for two matrices A and B
$(AB)^{-1} = B^{-1}, A^{-1}$

9. Find the dual basis of the basis set
$B = \int z_1 z_2 \int = \int 1+i, 1-i \int$
For the vector spaces V (R)

SECTION - B (खण्ड-ब)

(LONG ANSWER TYPE QUESTIONS) (दीर्घ उत्तरीय प्रश्न)

Note : Attempt any two questions. Each question carries 10.5 marks. **(10.5×2=21)**

1. If x and y are two vectors in an inner product space V, then
$\| x + y \|^2 + \| x - y \|^2 = 2\| x \|^2 + 2\| y \|^2$

2. Verify Cayley – Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

and find A^{-1}

3. Reduce the matrix

$$\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

to a ~~diagonal~~ form.

4. Prove that let V → U be and onto homomorphism with Ker T = W. Then there exists a one one onto mapping the subspaces of U and the subspace of V which contain W.