B.Sc. (Part-I) MATHEMATICS, 2009

Paper Third (Vector Analysis and Geometry)

Section – A : (Short Answer Questions)

Note : Attempt any seven questions. Each question has either 2 marks. (for B.A. students) or 3 marks (for B.Sc. students).

1. If $\bar{a}, \bar{b}, \bar{c}$ be three non-co-planar vectors, show that :

$$[\bar{a} \times \bar{b}\ \bar{b} \times \bar{c}\ \bar{c} \times \bar{a}] = [\bar{a}\ \bar{b}\ \bar{c}]^2.$$

2. If $\bar{a}', \bar{b}', \bar{c}'$ are reciprocal Vector of $\bar{a}, \bar{b}, \bar{c}$ respectively, then

$$\bar{a}' \times \bar{b}' + \bar{b}' \times \bar{c}' + \bar{c}' \times \bar{a}' = \frac{\bar{a} + \bar{b} + \bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]}.$$

3. If $\frac{d\bar{a}}{dt} = \bar{r} \times \bar{a}$ and $\frac{d\bar{b}}{dt} = \bar{r} \times \bar{b}$ then, show that $\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{r} \times (\bar{a} \times \bar{b})$.

4. Evaluate : $\int_S \bar{r}.\hat{n}\, ds$

where s is a closed surface.

5. Write the conditions when the general equation of the second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents different conics.

6. Find the equation of the normal of the point α to the conic

$$\frac{l}{r} = 1 + e\cos\theta.$$

7. Find the equation of plane through (1, 2, 3) and parallel to the plane $3x + 4y - 5z = 0$.

8. Prove that the lines $x = ay + b; z = cy + d$ and $x = a'y + b'; z = c'y + d'$ are perpendicular if $aa' + cc' = -1$.

9. Find the equation of the cone with vertex at the origin and passing through the curve

$ax^2 + by^2 = 2z; lx + my + nz = p.$

10. Find the equation of a right circular cylinder.

Section – B : (Long Answer Questions)

Note : Attempt any two questions. Each question has either 10 marks (for B. A. students) or 14 1/2 marks (for B.Sc. students).

11. (a) Prove that :

$$\text{curl}\ \frac{\bar{a} \times \bar{r}}{r^3} = -\frac{\bar{a}}{r^3} + \frac{3\bar{r}}{r^5}(\bar{a}\,.\,\bar{r})$$

where $\bar{a}$ is a constant vector.

(b) Evaluate :

$\int_C \bar{F}.d\bar{r}$, where $\bar{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and the curve C is the rectangle in the xy plane bounded by $y = 0; x = a; y = b; x = 0$.

12. State and prove Gauss divergence theorem.

13. Trace the parabola

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

and find the co-ordinates of its vertex, focus, and the equation to its directrix.

14. Find the condition that the general homogeneous equation of second degree in x, y, z, i.e

$$ax^2 + by^2 + cz^2 + 2hxy + 2fyz + 2gzx = 0$$

represents a pair of plane.