B.Sc. I – Mathematics (First Paper), 2006

(Algebra, Matrices and Trigonometry)

Note : 1. Attempt all section.

2. All objective type question of section A are compulsory.

3. Attempt any eight question from section B and any two questions from section C.

SECTION – A

1. If a, b and c are two elements of a group G, then $(abc)^{-1}$ is equal to :
(a) $a^{-1}b^{-1}c^{-1}$ (b) $c^{-1}b^{-1}a^{-1}$ (c) abc (d) cba

2. A non empty subset H of a group G is a subgroup of G iff $a, b \in H$:
(a) $ab^{-1} \in H$ (b) $a^{-1}b \in H$ (c) $b^{-1}a^{-1} \in H$ (d) $ab \in H$

3. The identity element of the group $(z, +)$ is :
(a) 1 (b) 2 (c) 3 (d) 0

4. If $f : (z, +) \longrightarrow (z, +)$ is a group homomorphism then $f(0) =$
(a) 1 (b) 0 (c) 2 (d) 3

5. The set of all natural numbers is :
(a) additive group (b) multiplicative group
(c) cyclic group (d) not a group

6. In a group $\{(1, \omega, \omega^2)\}$ where $\omega = 1^{1/3}$, the inverse of ω^2 is :
(a) 1 (b) ω (c) ω^2 (d) none of these

7. The rank of matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is :
(a) 1 (b) 2 (c) 3 (d) 4

8. A matrix $A = [a_{ij}]_{m \times n}$ is called a square matrix if :
(a) $m \le n$ (b) $m \ge n$ (c) $m = n$ (d) none of these

9. The value of $\tan(ix)$ is :
(a) $i \tanh x$ (b) $i \sinh x$ (c) $i \cosh x$ (d) $i \operatorname{sech} x$

10. The value of $\operatorname{cosech} x$:
(a) $\dfrac{2}{e^x - e^{-x}}$ (b) $e^x + e^{-x}$ (c) $\dfrac{e^x + e^{-x}}{2}$ (d) $\dfrac{2}{e^x + e^{-x}}$

SECTION – B

1. Prove that the set of cube roots of unity is abelion finite group with respect to multiplication.

2. Show that if every element of a group G is its own inverse, then G is abelion.

3. Show that any element of the group and its inverse have the same order.

4. Prove that the intersection of two subgroup is a subgroup.

5. If $f : G \to G'$ be a group homomorphism and e and e' be identities in G and G' respectively, then show that $f(e) = e'$.

6. If $a \in G$, G is any group, then show that

$$(a^{-1})^{-1} = a.$$

7. Prove that the matrix A is Hermitian, where

$$A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$$

8. If $A = [2 \quad 1]$, $B = \begin{bmatrix} 1 & -2 & 0 \\ 4 & 5 & -3 \end{bmatrix}$, then show that :

$$(AB)' = B'A'.$$

9. Show that the following matrix is unitary :

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

10. Find the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 4 & 10 & 18 \end{bmatrix}$$

11. Resolve $\tan(\alpha + i\beta)$ into real and imaginary part.

12. Find the general value of $\log(-3)$ *i.e.*, $\text{Log}(-3)$.

SECTION – C

1. Show that the set of all integers is a commutative ring with respect of addition and multiplication.

2. Prove that the relation of isomorphism of groups in the set of all groups is an equivalence relation.

3. Find characteristic roots and characteristic vectors of the matrix A, where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

4. If $u = \log_e \tan(\pi/4 + \theta/2)$, then prove that :

$$\sinh u = \tan \theta.$$