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BS(H)–1033

B. Sc. (M/S/C) (First Semester) EXAMINATION, 2020
PROBABILITY

Time : Three Hours
Maximum Marks : 70

Note : Attempt questions from both Sections as directed.

Section—A
(Short Answer Type Questions)

Note : Attempt any *seven* questions. Each question carries 4 marks.　　　　　7×4=28

1. For n events $A_1, A_2, ..., A_n$, prove that :

$$P\left(A_1 \cup A_2 \cup ...... \cup A_n\right) = \sum_{i=1}^{n} P(A_i)$$
$$- \sum_{\substack{i,j=1 \\ i \neq j}}^{n} P\left(A_i \cap A_j\right)$$
$$+ \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} P\left(A_i \cap A_j \cap A_k\right)$$
$$+ (-1)^{n-1} P\left(A_1 \cap A_2 \cap ...... \cap A_n\right)$$

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2. A random variable X has a mean value 3 and variance 2. What is the least value of $P\left[|X - 3| < 2\right]$?

3. Show that the covariance of two independent variates is equal to zero.

4. Find the chance of throwing 10 exactly in one throw with three dice.

5. Explain marginal and conditional distribution.

6. Find the mean for the following :

$$f(x) = \begin{cases} \dfrac{(x-1)^3}{4}, & 1 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

7. Define random experiment and trial sample space.

8. Discuss the effect of change of origin and scale on moment generating function.

9. Explain joint probability mass function of two random variables.

10. Explain Chebyshev's theorem.

Section—B

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 21 marks. $2 \times 21 = 42$

1. (a) Let $X_1, X_2, \ldots\ldots, X_n$ be mutually and iid random variables, with $E(X_i) = \mu$ and variance $(X_i) = \sigma^2 < \infty$. If $S_n = X_1 + X_2 + \ldots\ldots + X_n$, prove that the law of large numbers does not hold good for the sequence $\{S_n\}$.

    (b) Find the moment generating function of the random variable X having the probability density function :

    $$f(x) = \begin{cases} x, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$$

    Also find the mean and variance of X using moment generating function.

P. T. O.

2. (a) There are three similar coins, one of which is ideal and other two are biased. The chances of head are respectively 1/3 and 2/3. A coin is selected at random and tossed twice. If head occurs both times, find the probability that the ideal coin was selected. https://www.upadda.com

    (b) A husband and wife appear in an interview for two vacancies in the same post. The probability of the husband's selection is 1/7 and that of the wife's selection is 1/5. What is the probability that only one of them will be selected ?

3. (a) Let X be a continuous random variable with probability density function :

    $$f(x) = \begin{cases} ax; & 0 \le x \le 1 \\ a; & 1 \le x \le 2 \\ -ax + 3a; & 2 \le x \le 3 \\ 0; & \text{elsewhere} \end{cases}$$

    (i) Determine the constant a.

    (ii) Compute $P(X \le 1.5)$.

(b) If the joint distribution function of X and Y is given by :

$$F_{xy}(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)}; & x > 0 \\ & y > 0 \\ 0 & ; \text{ elsewhere} \end{cases}$$

(i) Find the marginal densities of X and Y.

(ii) Are X and Y independent ?

(iii) Find $P(X \le 1 \cap Y \le 1)$ and $P(X + Y \le 1)$.

4. (a) In a continuous distribution whose relative frequency density is :

$$f(x) = y_0 \, x(2 - x), \quad 0 \le x \le 2$$

(i) Show that the distribution is symmetrical.

(ii) Find mean deviation about mean.

P. T. O.

(b) A and B are two weak students in mathematics and their chances of solving a problem in mathematics correctly are 1/6 and 1/8 respectively. If the probability of their making a common error is 1/525 and they obtain the same answer, find the probability that their answer is correct.

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