B. A. / B. Sc. (Final) Examination 2010
MATHEMATICS
Paper : III
(Vector Analyis and Geometry)

Note : (i) Attempt questions from all the sections.

Section–A
(Short Answer Questions)

Note : Attempt any seven questions, Each question has 2 marks (for B. A. students), of 3 marks (for B. Sc. student).

1. For any three vectors $\bar{a}\ \bar{b}\ \bar{c}$ prove that.

$\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$

2. If $\bar{a}$, $\bar{b}$, $\bar{c}$ are reciprocal vectors of $\bar{a}$, $\bar{b}$, $\bar{c}$ respectively then prove

that $(\bar{a} \times \bar{b}) + \bar{b} \times \bar{c}) + \bar{c} \times \bar{a}) = \dfrac{\bar{a} + \bar{b} + \bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]}$

3. If $\bar{a} = 5\hat{i} + 3t^2\hat{j} + 2 t\ \hat{b}$ then find

$\dfrac{d\bar{a}}{dt}$ and $\dfrac{d^2\bar{a}}{dt^2}$

4. Prove that :–

$\text{div} [(\bar{r} \times \bar{a}) \times \bar{b}] = - 2\bar{a}\bar{b}$

where $\bar{a}$ and $\bar{b}$ are constant vectors.

5. Prove that :–

$\ddot{v}\ \left(\dfrac{\bar{r}}{r^3}\right) = 0$

6. Find the centre of the conic :–

$14x^2 - 4xy + 11 y^2 - 44x - 58 y + 71 = 0$

7. Find the equation to the hyperbola which has $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ as its asymptotes and passes through origin.

8. Find the equation of tangent at the point "α" on the conic $\dfrac{1}{r} = 1 + e \cos \theta$.

9. Show that the two straight lines are parallel to each other provided

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

Where a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of the two lines, respectively.

10. Find the equation to the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

Section-B
(Long Type Questions)

Note : Attempt any two questions, Each question has 10 marks (for B. A. students) and 14½ marks (for B. Sc students).

11. (a) Prove that

$$div\ grad\ r^m\ m\ (m+1)\ r^{m-2}$$

(b) Show that

$$curl\ \{\ (\bar{r}\times\bar{a})\times\bar{b}\}\ =\ \bar{b}\times\bar{a}$$

Where $\bar{a}$ and $\bar{b}$ are constant vectors

12. (a) State and prove shobes theorem.

13. Trace the curve

$$16x^2-24xy+9y^2-104x-172y+44=0$$

and find the coordinates of focus.

14. Find the centre and radius of the circle in which the sphere

$$x^2+y^2+z^2-8x+4y+8z-45=0\ is\ cut\ by\ the\ plane\ x-2y+2z=3\ \bullet$$